

# Properties of "35" Spin- $\frac{5}{2}$ Baryon Resonances in a Model with Broken $SU(3)$ <sup>†</sup>

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We investigate the properties of a set of  $J=\frac{5}{2}^+$  resonances appearing in a 35-dimensional representation of  $SU(3)$ , as proposed by Abers, Balázs, and Hara. A simple dynamical calculation gives an estimate for the mass differences within the supermultiplet. The matrix elements for the  $SU(3)$  allowed decays into meson plus resonance are given in terms of one parameter and the  $SU(3)$ -violating matrix elements for decay into meson plus baryon are given by two parameters.

**R**ECENTLY Abers, Balázs, and Hara<sup>1</sup> have studied  $\Pi\Delta$  ( $\Pi$  refers to the  $0^-$  meson octet and  $\Delta$  to the  $\frac{3}{2}^+$  baryon decuplet) scattering in a static model solved by the  $N/D$  method with linear  $D$  function and forces arising from baryon and baryon-resonance exchange. They predict a resonance in the  $J=I=\frac{5}{2}$  state, which in unitary symmetry would be a member of a supermultiplet containing 35 states.

A  $p\pi^+\pi^+$  peak has been observed at 1560 MeV.<sup>2</sup> It is not yet clear that this object can appropriately be identified with the  $(\frac{5}{2}, \frac{5}{2})$  resonance because (a) its spin has not yet been measured, and (b) it may be that this peak is a result of constructive interference of two  $N^{*++}$  resonances.<sup>2,3</sup> However, we will suppose here that the observed peak does correspond to the  $(\frac{5}{2}, \frac{5}{2})$  resonance and will use this fact to fix the average mass of the **35**, which is not predicted in a static-model calculation.

Our purpose in this note is to study the consequences of breaking  $SU(3)$  symmetry on the masses and decays of members of the **35**. Although our calculations are quite crude, it is hoped that they will provide a useful guide in analyzing experimental data pertaining to the **35** which, because there are many states a number of which are very broad, one can expect to be quite complex.

First, we will estimate the mass differences among the members of the **35**, arising from the medium-strong violations of  $SU(3)$ , using a method similar to that employed by Dashen and Frautschi<sup>4,5</sup> to calculate the mass splittings in the baryon octet and decuplet. We note that although the static model of Abers *et al.*<sup>1</sup> does not predict the average mass of the **35** we can combine this model with the  $S$ -matrix perturbation theory of

Ref. 5 to find the perturbations around a given average mass. In outline, the calculation is as follows.

Let us suppose that we have done an  $SU(3)$  symmetric dynamical ( $N/D$ ) calculation of the  $J=\frac{5}{2}^+$   $\Pi\Delta$  resonance, and consider first an  $SU(3)$  singlet perturbation; i.e., we change the masses of the baryon octet ( $B$ ), decuplet ( $\Delta$ ) and pseudoscalar mesons and the coupling constants in an  $SU(3)$  symmetric way and look for the resulting first-order change in the **35** mass. One finds

$$\delta M_{35} = A\delta M_B + B\delta M_\Delta + C\delta M_\Pi + (\text{coupling constant shifts, etc.}). \quad (1)$$

Applying the perturbation formulas of Ref. 5 and using the coupling constants and crossing matrices given by Abers *et al.*,<sup>1</sup> we find

$$A \approx -1, \quad |C| \ll |A|. \quad (2)$$

To find  $B$ , we recall that a bootstrap calculation cannot determine an absolute scale of mass, a fact which in the present (static) model implies that  $A+B \approx 1$  or  $B \approx 2$ . Finally, it turns out that the remaining terms in Eq. (1) (i.e., the effects of coupling shifts, and so on) are almost certainly small compared to the leading term,  $2\delta M_\Delta - \delta M_B$ , provided of course, that the model of Ref. 1 gives at least a qualitatively reliable account of the dynamics of the **35**.

If instead of making  $SU(3)$  singlet perturbations, we make perturbations in the baryon octet and decuplet masses transforming like an octet, it is a general result that the induced changes also follow an octet pattern.<sup>6</sup> We can parametrize these octet changes by the coefficients  $a$  and  $b$  that appear in the Gell-Mann-Okubo mass formula:

$$m = m_0 + aY + b[I(I+1) - \frac{1}{4}Y^2]. \quad (3)$$

For the particular case of the decuplet,  $[I(I+1) - \frac{1}{4}Y^2]$  is equal to  $(2 + \frac{3}{2}Y)$  and Eq. (3) reduces to  $m = m_0' + a'Y$  with  $m_0' = m_0 + 2b$  and  $a' = a + \frac{3}{2}b$ . Thus, recalling that the  $B$  and  $\Delta$  mass shifts are the dominant terms in the

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<sup>1</sup> E. S. Abers, L. A. P. Balázs, and Y. Hara, *Phys. Rev.* **136**, B1382 (1964).

<sup>2</sup> G. Goldhaber, Lecture at the Conference on Particle and High Energy Physics at Boulder, Colorado, 1964 (unpublished).

<sup>3</sup> F. E. Low and A. C. Zemach (private communication).

<sup>4</sup> R. F. Dashen and S. C. Frautschi (to be published).

<sup>5</sup> R. F. Dashen and S. C. Frautschi, *Phys. Rev.* **135**, B1190 (1964).

<sup>6</sup> S. L. Glashow, *Phys. Rev.* **130**, 2132 (1963).

present model for the **35** mass shifts, we can write

$$\begin{aligned} a_{35} &\approx A^{aa}a_B + A^{ab}b_B + B^{aa'}a_{\Delta'}', \\ b_{35} &\approx A^{ba}a_B + A^{bb}b_B + B^{ba'}a_{\Delta'}', \end{aligned} \quad (4)$$

which is the analog of Eq. (1).

In a simple case such as the present one where we treat only a single channel and one-particle exchange, one can use group theory<sup>7</sup> to relate the coefficients  $A^{ij}$  and  $B^{ij}$  of Eq. (4) to the  $A$  and  $B$  of Eq. (1). One finds

$$\begin{aligned} A^{aa} &= -\frac{1}{5}A^{ab} = -\frac{1}{8}, & B^{aa'} &= \frac{7}{16}, \\ A^{ba} &= -A^{bb} = \frac{1}{4}, & B^{ba'} &= \frac{1}{8}. \end{aligned} \quad (5)$$

Thus, putting in experimental values for  $a_B$ ,  $b_B$ , and  $a_{\Delta'}$ , and using the coefficients of Eq. (5) we obtain

$$a_{35} \approx -180 \text{ MeV}; \quad b_{35} \approx 20 \text{ MeV}. \quad (6)$$

While our numerical predictions of  $a_{35}$  and  $b_{35}$  are to a considerable extent model-dependent,<sup>8</sup> we know from rather general dynamical arguments that the **35** mass splitting must be dominantly octet (i.e., obey the Gell-Mann-Okubo sum rule). To see this, let us consider, for example, the possibility of a large **27** component in the **35** mass splittings. A **27** component in the **35** mass splittings can come only from **27**-type splittings of the  $\Pi$ ,  $B$ , or  $\Delta$  masses<sup>6</sup> (or possibly corrections to couplings which transform like part of a **27**). But the  $\Pi$ ,  $B$ , and  $\Delta$  mass splittings have a very small **27** component, which will lead to a small **27** component of the mass splitting in the **35** unless the latter reacts on itself in such a way as to enhance violations<sup>4</sup> of  $SU(3)$  which transform like part of a **27**. However, from the crossing matrix of Abers *et al.*,<sup>1</sup> one sees that the **35** has only a weak reaction on itself so we conclude that the **27** component of the **35** mass differences is small.

Glashow has conjectured<sup>6</sup> that the coefficients  $a$  and  $b$  which appear in the Gell-Mann-Okubo mass formula are the same for all supermultiplets with the same baryon number. One will note that our values for  $a_{35}$  and  $b_{35}$  are numerically very close to the values of  $a$  and  $b$  for the baryon octet.

Finally, determining  $m_0$  so that the  $I=\frac{5}{2}$ ,  $Y=1$  member of the **35** has the mass (1560 MeV) of the observed  $p\pi^+\pi^+$  peak assuming it is a  $J=I=\frac{5}{2}$  state, we obtain the masses given in Table I, where we also list the decomposition of the **35** into  $(Y, I)$  states.

Neglecting violations of  $SU(3)$ , the allowed decays of the (unstable) members of the **35** into  $\frac{3}{2}^+$  baryon resonance  $+\Pi$  will depend on one parameter  $\gamma$ . We

define a (partial) width as

$$\Gamma = |M|^2 \rho. \quad (7)$$

Here,  $\rho$  is a factor which includes phase-space and orbital-angular-momentum barrier factors. A possible choice for  $\rho$  would be

$$\rho \propto q(q^2/(q^2+\mu^2))^l, \quad (8)$$

$\mu$  being an appropriate interaction radius.<sup>9</sup> The matrix element  $|M|^2 = |\gamma|^2 \times |\text{Clebsch-Gordan coefficients}|^2$ . The Clebsch-Gordan coefficients for coupling **8** and **10** to **35** are taken from the table of McNamee and Chilton.<sup>10</sup> In Table I we give the squares of these matrix elements, summed over charge states.

Since the baryons and pseudoscalar mesons belong to octets and **8** $\otimes$ **8** does not contain the **35**, the decay of a member of the **35** into  $B+\Pi$  can proceed only through violations of  $SU(3)$ .

Assuming that violations of  $SU(3)$  transform like the eighth component of an octet, and noting that **8** $\otimes$ **10** and **8** $\otimes$ **27** each contain **35** once, while **8** $\otimes$ **8** and **8** $\otimes$ **10** do not contain **35** at all, we can obtain the  $SU(3)$  violating matrix elements in terms of two (possibly complex) parameters,  $\alpha$  and  $\beta$ . These matrix elements, again summed over all charge modes, are listed in Table I. Note that in some cases the matrix elements correspond to channels that according to our calculations would be closed.

We shall close with a list of comments pertaining to the results in Table I.

(1) According to our calculation, the  $Y=2$ ,  $I=2$  and  $Y=-3$ ,  $I=\frac{1}{2}$  members of the **35** are stable against strong decays.

(2) We expect  $\alpha$  and  $\beta$  to be  $\approx 30\%$  of  $\gamma$  in the amplitude, on the basis of the general order of magnitude of violations of  $SU(3)$ .

(3) The  $Y=-2$ ,  $I=0$  member of the **35** presumably decays through a pure violation of  $SU(3)$ .

(4) There are a few cases where we might expect the  $SU(3)$ -violating decays to be competitive with the  $SU(3)$ -allowed decays. For example, the  $(\pi N)$  decay of the  $Y=1$ ,  $I=\frac{3}{2}$  state may be comparable to the  $\pi\Delta$  mode. Also the coefficient for the decay of the  $Y=-1$ ,  $I=\frac{1}{2}$  object into  $\pi\Xi^*$  is only  $|\gamma|^2/16$ , so one might expect some of the  $SU(3)$ -violating decays into  $\bar{K}\Lambda$ ,  $\bar{K}\Sigma$ , and  $\pi\Sigma$  to be comparable to the  $\pi\Xi^*$  mode.

(5) The width of the  $Y=1$ ,  $I=\frac{5}{2}$  state is estimated by Abers *et al.*<sup>1</sup> to be  $\approx 200$  MeV, which seems to be compatible with experiment.<sup>2</sup>

(6) The width of the  $Y=-1$ ,  $I=\frac{3}{2}$  state will be  $\approx 100$  MeV, since the  $Q$  value for the  $\pi\Xi^*$  decay is about the same as that of the  $Y=1$ ,  $I=\frac{5}{2}$  state, whereas  $|M|^2$  is  $\approx |\gamma|^2/2$  compared to  $|\gamma|^2$ .

<sup>9</sup> S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).

<sup>10</sup> P. McNamee and F. Chilton, Rev. Mod. Phys. **36**, 1005 (1964).

<sup>7</sup> R. E. Cutkosky and P. Tarjanne, Phys. Rev. **132**, 1355 (1963).

<sup>8</sup> In our model, as in Ref. 1, we have used static kinematics and neglected many forces such as vector-meson exchange. The use of static kinematics in the calculation of mass differences within  $SU(3)$  supermultiplets appears to be a reasonable approximation [R. Dashen and S. Frautschi (to be published)]. We have not investigated the effect of vector-meson exchange in detail. However, there seems to be no reason why the vector-meson exchange forces should have more than the small effect that they seem to have in other static-model problems.

TABLE I. In this table, the masses and decomposition into  $(Y, I)$  states of the members of the 35 are given. Also given are relative probabilities for  $SU(3)$ -allowed and (strong)  $SU(3)$ -violating decays of members of the 35. The threshold for each decay mode is indicated; where our calculation indicates that the channel is closed we enclose that decay mode in a bracket.

Particle ( $Y, I$ )	Predicted mass (MeV)	$ M ^2$ , mode	$SU(3)$ -allowed decays		$ M ^2$ , Strong $SU(3)$ -violating decays	
			Threshold (MeV)	$ M ^2$	Mode	Threshold (MeV)
(2, 2)	1310	$[K\Delta]$	1730	$ \gamma ^2$	None	
(1, $\frac{3}{2}$ )	1460	$\pi\Delta$	1375	$\frac{1}{16} \gamma ^2$	$\pi N$	1180
		$[\eta\Delta]$	1785	$\frac{5}{16} \gamma ^2$	$[K\Sigma]$	1690
		$[K\Sigma^*]$	1875	$\frac{5}{8} \gamma ^2$		$\frac{5}{32} \alpha-\beta ^2$
(1, $\frac{5}{2}$ )	1560	$\pi\Delta$	1375	$ \gamma ^2$	None	
(0, 1)	1610	$\pi\Sigma^*$	1520	$\frac{1}{12} \gamma ^2$	$\pi\Lambda$	1250
		$[\bar{K}\Delta]$	1730	$\frac{1}{12} \gamma ^2$	$\pi\Sigma$	1330
		$[\eta\Sigma^*]$	1930	$\frac{1}{2} \gamma ^2$	$\bar{K}N$	1435
		$[K\Sigma^*]$	1930	$\frac{1}{3} \gamma ^2$	$[\eta\Sigma]$	1740
					$[K\Sigma]$	1815
(0, 2)	1690	$\pi\Sigma^*$	1520	$\frac{3}{4} \gamma ^2$	$\pi\Sigma$	1330
		$[\bar{K}\Delta]$	1730	$\frac{1}{4} \gamma ^2$		$\frac{3}{4} \beta ^2$
(-1, $\frac{1}{2}$ )	1760	$\pi\Sigma^*$	1670	$\frac{1}{16} \gamma ^2$	$\pi\Sigma$	1460
		$[K\Sigma^*]$	1875	$\frac{1}{4} \gamma ^2$	$\bar{K}\Lambda$	1610
		$[\eta\Sigma^*]$	2080	$\frac{9}{16} \gamma ^2$	$\bar{K}\Sigma$	1690
		$[K\Omega^-]$	2180	$\frac{1}{8} \gamma ^2$	$[\eta\Sigma]$	1870
(-1, $\frac{3}{2}$ )	1820	$\pi\Sigma^*$	1670	$\frac{1}{2} \gamma ^2$	$\pi\Sigma$	1460
		$[\bar{K}\Sigma^*]$	1875	$\frac{1}{2} \gamma ^2$	$\bar{K}\Sigma$	1690
(-2, 0)	1910	$[\bar{K}\Sigma^*]$	2025	$\frac{1}{2} \gamma ^2$	$\bar{K}\Sigma$	1815
		$[\eta\Omega^-]$	2235	$\frac{1}{2} \gamma ^2$		$\frac{1}{2} \alpha ^2$
(-2, 1)	1950	$\pi\Omega^-$	1825	$\frac{1}{4} \gamma ^2$	$\bar{K}\Sigma$	1815
		$[\bar{K}\Sigma^*]$	2025	$\frac{3}{4} \gamma ^2$		$\frac{3}{4} \beta ^2$
(-3, $\frac{1}{2}$ )	2080	$[\bar{K}\Omega^-]$	2180	$ \gamma ^2$	None	

(7) The width of the  $Y = -1, I = \frac{1}{2}$  state is presumably smaller than that of the  $Y = 1, I = \frac{5}{2}$  state by an order of magnitude or more, because its  $Q$  value is smaller and its matrix element for  $\pi\Sigma^*$  decay is only  $\approx \frac{1}{16}$  that for the  $\pi\Delta$  decay of the  $Y = 1, I = \frac{5}{2}$  state.

(8) The  $Y = -1, I = \frac{1}{2}$  resonance at 1760 MeV and the  $Y = -1, I = \frac{3}{2}$  resonance at 1820 MeV will have a considerable overlap, and analysis of the decays will be difficult because of interference effects.

In this regard, it is very interesting to consider a recent experiment of Smith *et al.*,<sup>11</sup> who find a peak (position  $\approx 1810$  MeV, width  $\approx 70$  MeV) in  $Kp$  scattering which may be a  $\pi\Sigma^*$  resonance. Its spin-parity is probably  $\frac{5}{2}^+$  or  $\frac{3}{2}^-$ . Let us suppose it is the former. Then this might be our  $Y = -1, I = \frac{3}{2}$  state at 1820 MeV with an estimated width of  $\approx 100$  MeV. However, Smith *et al.*<sup>11</sup> find that an isospin of  $\frac{1}{2}$  is favored, and an

appreciable fraction of decays into  $\bar{K}\Lambda$ . We note that the  $Y = -1, I = \frac{1}{2}$  state lies nearby and, according to Table I, may well have an appreciable probability for decay into  $\bar{K}\Lambda$ . A more detailed investigation of this resonance on the assumption that one has a mixture of isospin states would seem worthwhile.

(9) The  $Y = 0, I = 2$  state at 1670 MeV has a width of  $\approx 100$ –200 MeV, while the  $Y = 0, I = 1$  state has a much smaller width because its matrix element into the only open channel ( $\pi\Sigma^*$ ) is small ( $|\gamma|^2/12$ ). Again, we can expect considerable interference of these resonances.

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<sup>11</sup> G. A. Smith, J. S. Lindsey, J. J. Murray, J. Button-Shafer, A. Barbaro-Galtieri *et al.*, Phys. Rev. Letters 13, 61 (1964).